

Tribhuvan University
Institute of Science and Technology
2066

Bachelor Level/ First Year/ Second Semester/ Science

Computer Science and Information Technology (MTH. 155)

(Linear Algebra)

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

Group A

(10x2=20)

1. When is a system of linear equations consistent or inconsistent?
2. Write numerical importance of partitioning matrices.
3. How do you distinguish singular and non-singular matrices?
4. If A and B are $n \times n$ matrices, then verify with an example that $\det(AB) = \det(A) \det(B)$.
5. Calculate the area of the parallelogram determined by the columns of $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$.
6. Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is in $\text{Nul}(A)$, where, $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.
7. Determine if $\{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
8. Find the characteristic polynomial and the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.
9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.
10. Let $\{u_1, \dots, u_p\}$ be an orthogonal basis for a subspace W of $\mathbf{R}^n$. Then prove that for each $y \in W$, the weights in $y = c_1 u_1 + \dots + c_p u_p$ are given by

$$c_j = \frac{y \cdot u_j}{u_j \cdot u_j} \quad (j = 1, \dots, p)$$

Group B

(5x4=20)

11. Prove that any set $\{v_1, \dots, v_p\}$ in $\mathbf{R}^n$ is linearly dependent if $p > n$.
12. Consider the Leontief input-output model equation $x = cx + d$, where the consumption matrix

is

$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}.$$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.

13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State and prove the unique representation theorem for coordinate system.

14. What do you mean by eigen values, eigen vectors and characteristic polynomial of a matrix?

Explain with suitable examples.

15. Define a Gram-Schmidt process. Let $W = \text{span}\{x_1, x_2\}$, where

$$x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

Then construct an orthogonal basis $\{v_1, v_2\}$ for W .

Group C

(5x8=40)

16. Given the matrix $\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & -15 \end{bmatrix}$, discuss the forward phase and backward phase of the row reduction algorithm.

17. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, by using elementary row reduce the augmented matrix.

18. What do you mean by change of basis in $\mathbf{R}^n$? Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for $\mathbf{R}^2$ given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$. Find the change of coordinated matrix from B to C.

19. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, if possible.

OR

Find the eigen values of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$, and find the basis for each eigen space.

20. Find a least-squares solution for $Ax = b$ with

$$A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}.$$

What do you mean by least-squares problem?

OR

Define a least-squares solution of $Ax = b$, prove that the set of least squares solutions of $Ax = b$ coincides with the nonempty set of solutions of the normal equations $A^T Ax = A^T b$.